* For function, f

$\| f \|_1 = \int_a^b |f| \, dt$

$\| f \|_2 = \left( \int_a^b |f|^2 \, dt \right)^{1/2}$

$\| f \|_\infty = \max \{ |f(t)| \}$

Q) $W \subseteq V$ , $W^\perp \subseteq V$

$V = W \oplus W^\perp$

Let $V$ is an n-dimensional vector space.

Proof:- $\dim(V) = n$

Let us assume that $\dim(W) = r \leq n$.

Let $\{w_1, w_2, \ldots, w_r\}$ is an orthogonal basis for $W$

i.e. $W = [\{w_1, w_2, \ldots, w_r\}]$

$\{w_1, w_2, \ldots, w_r, w_{r+1}, \ldots, w_n\}$ is a basis for V.

$W^\perp = [\{w_{r+1}, \ldots, w_n\}]$

* Prove that if $S \subset V$ then $S \subseteq S^{\perp\perp}$

Proof:- Let $u \in S$ and $v \in S^\perp$

$\langle u, v \rangle = 0$

$\Rightarrow \langle v, u \rangle = 0$

$\Rightarrow u \in (S^\perp)^\perp$

$\therefore S \subseteq (S^\perp)^\perp$