* Let X_1 and X_2 be two subspaces of a v.s V, then the sum of X_1 and X_2 is defined as,

$$X_1 + X_2 = \{ u+v \mid u \in X_1 \text{ and } v \in X_2 \}$$

* $X_1 + X_2$ is a subspace

let $u_1 + v_1 \in X_1 + X_2$

$u_2 + v_2 \in X_1 + X_2$

Now, $\alpha(u_1+v_1) + \beta(u_2+v_2) = \alpha u_1 + \alpha v_1 + \beta u_2 + \beta v_2$

$= (\alpha u_1 + \beta u_2) + (\alpha v_1 + \beta v_2)$

$\in X_1 + X_2$ $[\because \alpha u_1 + \beta u_2 \in X_1$ & $\alpha v_1 + \beta v_2 \in X_2]$

$\therefore$ $X_1 + X_2$ is a subspace of V.

* Prove that $[X_1 \cup X_2]$ is a subspace of V.

* Show that $X_1 + X_2 = [X_1 \cup X_2]$

1) Let $X_1 = \{u_1, u_2, \ldots, u_n\}$

$X_2 = \{v_1, v_2, \ldots, v_n\}$

$X_1 \cup X_2 = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$

$[X_1 \cup X_2] = [u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n]$

Let $u, v \in [X_1 \cup X_2]$ such that

$u = \alpha_1 u_1 + \alpha_2 u_2 + \cdots + \alpha_n u_n + \beta_1 v_1 + \beta_2 v_2 + \cdots + \beta_n v_n$

$v = c_1 u_1 + c_2 u_2 + \cdots + c_n u_n + d_1 v_1 + d_2 v_2 + \cdots + d_n v_n$,

where $\alpha_1, \ldots, \alpha_n, \beta_1, \ldots, \beta_n, c_1, \ldots, c_n, d_1, \ldots, d_n$ are scalars.