∴ z is not uniquely expressible which is a contradiction.

Theorem:- Dt:- 22.01.2020

In a vectorspace V suppose $\{v_1, v_2, \ldots, v_n\}$ is an ordered set of vectors with $v_1 \neq 0$. The set is LD iff one of the vectors $v_2, v_3, \ldots, v_k \in [v_1, v_2, \ldots, v_{k-1}]$ for some $k = 2, 3, \ldots$

Proof:- S.P Let $v_k \in [v_1, v_2, \ldots, v_{k-1}]$

Claim:- $\{v_1, v_2, \ldots, v_k\}$ is LD.

$v_k \in [v_1, v_2, \ldots, v_{k-1}]$

$\Rightarrow v_k = \alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_{k-1} v_{k-1}$

$\Rightarrow v_k$ is the linear combination of $v_1, v_2, \ldots, v_{k-1}$

Hence $\{v_1, v_2, \ldots, v_{k-1}, v_k\}$ is LD.

N.P Let the set $\{v_1, \ldots, v_k, \ldots, v_n\}$ is LD.

Assume that $S_1 = \{v_1\}$

$S_2 = \{v_1, v_2\}$

$\vdots$

$S_{k-1} = \{v_1, v_2, \ldots, v_{k-1}\}$

$S_k = \{v_1, v_2, \ldots, v_k\}$

$\vdots$

$S_n = \{v_1, v_2, \ldots, v_n\}$

∴ S_1 is LI.