

Matrix:-

Matrix is a rectangular array of $m \times n$ in which we can arrange mn well-defined elements.

$$A = [a_{ij}] \quad \begin{matrix} i = 1, 2, \dots, m \\ j = 1, 2, \dots, n \end{matrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}_{m \times n}$$

→ Determinant is a scalar which is associated with a square matrix and is denoted by $|A|$ or $\det(A)$.

→ $|A| = 0$, then A is singular matrix

$|A| \neq 0$, then A is non-singular matrix

Q:- Find rank of

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \\ 2 & 3 & 1 & 1 \\ 0 & 2 & -1 & -1 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - R_1$$

$$R_3 \rightarrow R_3 - 2R_1$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & -1 & -5 & -7 \\ 0 & 2 & -1 & -1 \end{bmatrix}$$

$$R_1 \rightarrow R_1 + 2R_2$$

$$R_3 \rightarrow R_3 - R_2$$

$$R_4 \rightarrow R_4 + 2R_2$$

$$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & -3 & -4 \\ 0 & 0 & -5 & -7 \end{bmatrix}$$

$$R_4 \rightarrow R_4 - 5R_3$$

$$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & -3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$