4) Let $u_1 = (x_1, y_1, z_1)$

$u_2 = (x_2, y_2, z_2)$

$$T(\alpha u_1) = T(\alpha x_1, \alpha y_1, \alpha z_1) = (\alpha x_1 + \alpha y_1 + \alpha z_1, 1)$$

$$\neq \alpha T(u_1) = \alpha(x_1 + y_1 + z_1, 1) = (\alpha x_1 + \alpha y_1 + \alpha z_1, \alpha)$$

Results:-

Let $T: U \to V$ be a linear map, then

(i) $T(0_U) = 0_V$ (converse need not true)

(ii) $T(-u) = -T(u)$

(iii) $T(\alpha_1 u_1 + \alpha_2 u_2 + \dots + \alpha_n u_n) = \alpha_1 T(u_1) + \dots + \alpha_n T(u_n)$.

(i) Let $u \in U$

$\Rightarrow -u \in U$

$u + (-u) = 0$

$$T(0) = T(u + (-u)) = T(u) + T(-u)$$

$$= T(u) - T(u)$$

$$= 0$$

(ii) $T(-u) = T\{(-1)\cdot u\} = (-1)T(u)$ [$\because T$ is linear]

$= -T(u)$.

(iii) $T(\alpha_1 u_1 + \alpha_2 u_2 + \dots + \alpha_n u_n) = T\{\alpha_1 u_1 + (\alpha_2 u_2 + \dots + \alpha_n u_n)\}$

$= T(\alpha_1 u_1) + T(\alpha_2 u_2 + \dots + \alpha_n u_n)$

$= \alpha_1 T(u_1) + T\{(\alpha_2 u_2) + (\alpha_3 u_3 + \dots + \alpha_n u_n)\}$

$= \alpha_1 T(u_1) + T(\alpha_2 u_2) + T(\alpha_3 u_3 + \dots + \alpha_n u_n)$

$= \alpha_1 T(u_1) + \alpha_2 T(u_2) + T(\alpha_3 u_3 + \dots + \alpha_n u_n)$

$\vdots$

continuing in this way we get

$= \alpha_1 T(u_1) + \alpha_2 T(u_2) + \dots + \alpha_n T(u_n)$