

$$V = P_3$$

$$U = \{ p \in P_3 \mid p(2) = 0 \}$$

$$W = \{ p \in P_3 \mid p'(1) = 0 \}$$

Find, $U, W, U \cap W, U + W$.

HW

Q:- U and W are two distinct subspaces of a V s.t. V and $\dim(V) = 7$, $\dim(U) = 5$, $\dim(W) = 4$.

Then find the all possible dimension of $U \cap W$.

Result:-

A linear transformation T is completely determined by its values on the elements of a basis.

If $B = \{u_1, u_2, \dots, u_n\}$ is a basis of U and

$v_1, v_2, \dots, v_n$ vectors (not necessarily distinct) in V ,

then $\exists$ a unique linear transformation, $T: U \rightarrow V$ st

$$T(u_i) = v_i, \quad i = 1, 2, \dots, n \quad \text{--- (A)}$$

* Let $u \in U$, then u can be uniquely expressed as

$$u = \alpha_1 u_1 + \alpha_2 u_2 + \dots + \alpha_n u_n$$

$$T(u) = \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n$$

Now, we will prove that (i) T is linear

(ii) T satisfies the eq (A)

(iii) T is unique.